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Is the empirical approximation $Y/G \approx$ constant applicable to high-pressure and high-temperature environments for metals?

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Abstract
Recently, we have found, by means of a shock wave experiment, that an empirical relation $Y/G \approx 1.9 \times 10^{-2}$ (Y is the yield strength and G is the shear modulus) is applicable for describing the strength effect for shocked 93W (93% W with 7% Fe–Ni–Co as binder) in the pressure range up to 150 GPa. This represents an extension of existing knowledge of the empirical approximation $Y/G \approx$ constant for potassium obtained at liquid-N_2 temperature and in the pressure range below 0.55 GPa. This approximation is advantageous in allowing one to simply and conveniently construct the constitutive equation for shocked metals.

1. Introduction

In 1980, Steinberg, Cochran, and Guinan (SCG) [1] proposed a constitutive model for describing the behaviour of shocked metals under high pressure loadings, as functions of pressure p and temperature T; Y and G can be written as

$$G = G_0[1 + (G'_p/G_0)(p/\eta^{1/3}) + (G'_T/G_0)(T - 300)] \quad (1)$$

$$Y = Y_0(1 + \beta\varepsilon)^n[1 + (Y'_p/Y_0)(p/\eta^{1/3}) + (Y'_T/Y_0)(T - 300)] \quad (2)$$

subject to the limitation that

$$Y_0(1 + \beta\varepsilon)^n \leqslant Y_{max} \quad (3)$$

which is valid for $\varepsilon \geqslant 10^5$ s^{-1} or above ~10 GPa, where $\eta = v_0/v$ is the compression ratio, v is the specific volume, β and n are two work-hardening parameters, ε is the effective plastic strain,

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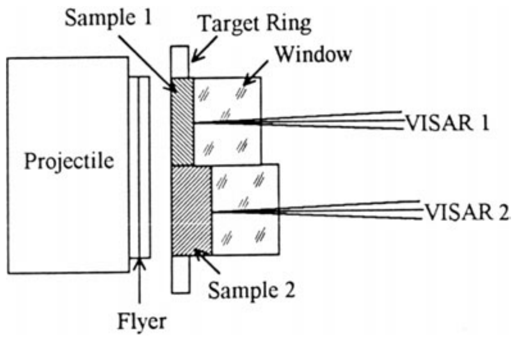

Figure 1. A schematic diagram of the experimental set-up.

Y_{max} is the largest value of Y in the literature, the subscripts refer to the initial or reference state, primed parameters with the subscripts p and T indicate first-order partial derivatives with respect to p and T, respectively, at the reference state. Obviously, equations (1) and (2) are two Taylor's expansions for G and Y but truncated at second-order derivatives, except the work-hardening term in the Y-expression.

In general, values of G_0, G'_p, and G'_T may be measured by supersonic measurement under hydrostatic pressure conditions, despite rather larger uncertainties always appearing in determinations of G'_p and G'_T since the upper pressure and temperature are commonly below $\sim$10 GPa and $\sim 10^3$ K. Also, accurate determinations of Y'_p and Y'_T are more difficult to achieve than those of G'_p and G'_T. For the latter case, SCG used an empirical approximation:

$$Y/G \approx \text{ constant} \qquad (4)$$

which was first reported by Chua and Ruoff [2] for potassium, and is useful for finding replacements for Y'_p and Y'_T. Then one may obtain

$$Y'_p/Y_0 \approx G'_p/G_0, \qquad Y'_T/Y_0 \approx G'_T/G_0. \qquad (5)$$

SCG used equation (5) in hydrodynamic simulations for several shocked metals, and achieved some success. Nevertheless, we still do not know whether equation (4) is also applicable to the high-p and high-T region. Therefore, it seems that directly measuring the values of G and Y and testing the validity of equation (5) at high p and high T are tasks needing to be undertaken.

2. Experimental assembly and results

93W was used as a model material. The experimental arrangement is shown schematically in figure 1 [4]. Shock (or so-called preshock)/reshock and shock/release experiments were conducted. In a shock/reshock experiment, the shock impedance of the front-flyer should be lower than that of the back-flyer. Upon impact on the sample of the flyer, a shock wave propagates in the sample and a forward left shock will be produced in the front-flyer, which will again be reflected from the front/back-flyer interface as a shock wave from reshocking the preshocked sample. If the shock impedance of the front-flyer is higher than that of the back-flyer, a shock/release experiment is performed. A stepped sample is used for the convenience of being able to directly determine the longitudinal wave velocities from VISAR's records (including the longitudinal elastic and plastic or bulk velocity of the sample at each given particle velocity).

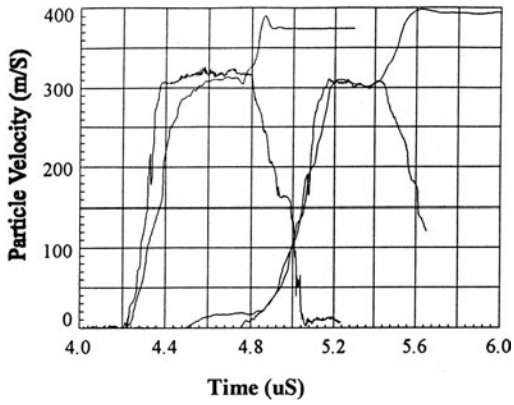

Figure 2. The interface velocity profile at the window–sample interface for impact velocity 600 m s^{-1}.

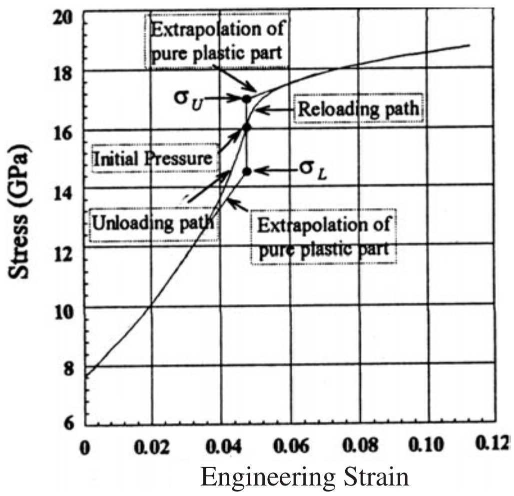

Figure 3. Experimental stress–strain paths (impact velocity 600 m s^{-1}; the inferred $Y = 1.84$ GPa).

In the data processing, a three-step procedure is utilized:

(1) Sample/window interface velocity (u_w) versus time (t) curves are deduced from VISAR's original records [5] (see figure 2).
(2) Lagrangian sound velocities (C) for each given particle velocity are computed from the u_w–t curves deduced, and then one uses them to compute the corresponding stress (σ_s) versus engineering strain (e) curve by the impedance-matching method [6] (see figure 3).
(3) The plastic segment of the σ_s–e curve deduced is determined by inspection; it is then extrapolated to the preshocked state e_0 (see figure 3). Thus, Y is obtained from the relation $Y = 3(\sigma_U - \sigma_L)/4$ [3, 6].

In general, the calculated C, obtained from the second step, may be for longitudinal elastic or plastic (or bulk) waves depending on which segment (elastic or plastic) the given particle velocity lies in. In this work, we choose the first arrival release or reshock around shock stress

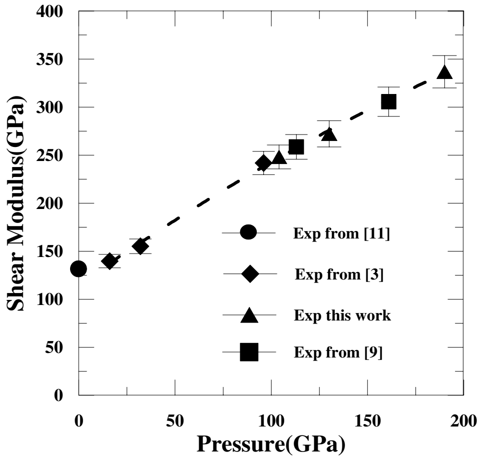

Figure 4. The shear modulus versus shock stress plot for 93W.

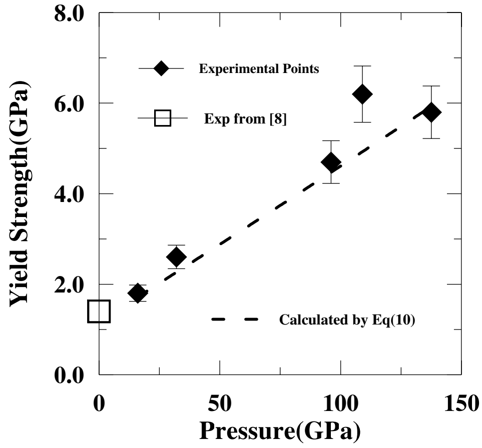

Figure 5. The yield strength versus stress plot for 93W.

Table 1. Experimental values of G and Y for 93W.

State	Stress (GPa)	0	16	32	96	104	113	130	161
e_0	Strain	0	0.05	0.089	0.193	0.210	0.213	0.236	0.256
G (GPa)		132[d]	139.7[b]	155.2[b]	214.9[b]	248.3	258.7[b]	272.3	305.8[b]
Y (GPa)		1.4[a]	1.8[b]	2.6[b]	4.8[b]	6.2	—	5.8	—

[a] From [8].
[b] From [3].
[c] From [9].
[d] From [11].

e_0 (see figure 3) as the longitudinal elastic one, with a velocity C_1 that can be computed through use of the equation

$$C_1 = (1/\rho)(1 - e_0)(\mathrm{d}\sigma_s/\mathrm{d}e)_0 \tag{6}$$

where ρ is the density. Therefore the corresponding shear modulus G may be computed from the following equation [7]:

$$G = 4\rho(C_e^2 - C_b^2)/3 \tag{7}$$

where

$$C_b = C_0(1 - e_0)\sqrt{\frac{1 + \lambda e_0 - \gamma_0\lambda e_0^2}{(1 - \lambda e_0)^3}}. \tag{8}$$

C_b is the bulk wave velocity at the e_0-state, c_0 and λ are two material constants appearing in the linear D–u relation $D = C_0 + \lambda u$ (D is the shock wave velocity and u the particle velocity).

The experimental values of G and Y are listed in table 1 and shown in figures 4 and 5.

3. Discussion

3.1. Discussion of the validity of $Y/G \approx$ constant along the Hugoniot

The linear least-squares fit of $G \sim \sigma_0$ is

$$G = 110.4 + 2.18\sigma_0 - 7.62 \times 10^{-3}\sigma_0^2, \tag{9}$$

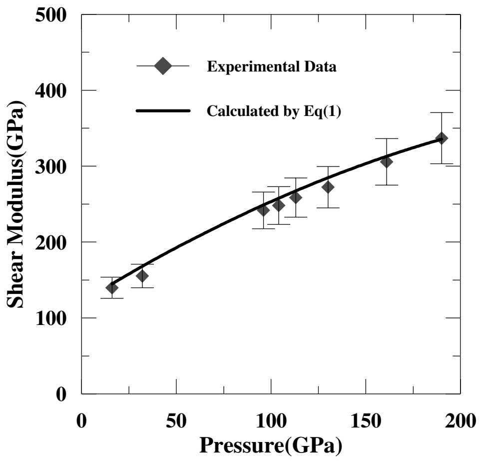

Figure 6. Comparison between the experimental data on G and the values calculated using equation (1) for shocked 93W.

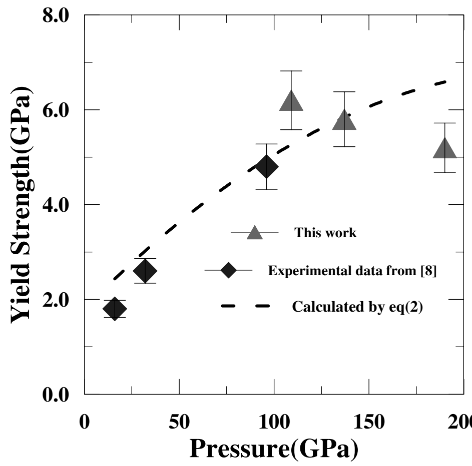

Figure 7. Comparison between the experimental data on Y and the values calculated using equation (2) for shocked 93W.

which is also plotted (as a dotted line) in figure 4, where both G and σ_0 are in GPa. To verify the validity of $Y/G \approx$ constant along the Hugoniot, we transform equation (4) to

$$Y = (Y_0/G_0)(110.4 + 2.18\sigma_0 - 7.62 \times 10^{-3}\sigma_0^2), \tag{10}$$

and we have drawn this in figure 5 as a dotted line. The result demonstrates that equation (10) is in comparatively good accord with the measurement data. A better fit would be expressed as

$$Y/G \approx 1.9 \times 10^{-2}. \tag{11}$$

3.2. *Discussion of the validity of $Y/G \approx$ constant off the Hugoniot*

In practice, the SCG model is used to carry out simulations involving both Hugoniot states and off-Hugoniot states. For this reason, two problems have been examined and are described in the following.

(1) Validity of the SCG constitutive equations (1) and (2) along the Hugoniot. Values of the material parameters in equations (1) and (2) were given in [11]. They are: $G_0 = 132$ GPa, $G'_p = 1.794$, $G'_T = -0.04$ GPa °C^{-1}, $Y_0 = 1.4$ GPa, $\beta \approx 1.3$, $\eta \approx 0.1$. We used them to calculate $G(\sigma_0)$ and $Y(\sigma_0)$ and then plotted the results (as solid and dotted curves, respectively) in figures 6 and 7. On comparing with experiments, it is found that equations (1) and (2) can reproduce the measured G and Y comparatively faithfully.

(2) Validity of the SCG constitutive equations (1) and (2) off the Hugoniot. Two hydrodynamic simulation examples are given in figures 8 and 9. On comparing to the experimental u_w–t profile, it can be seen that equations (1) and (2) are indeed appropriate not only to Hugoniot states, but also to off-Hugoniot states to a satisfactory degree. As regards the significant deviation from the experiment profile appearing in the release tail calculation (see figure 9), which also emerged in the SCG simulation [1], the reason for this finding has been not yet been found.

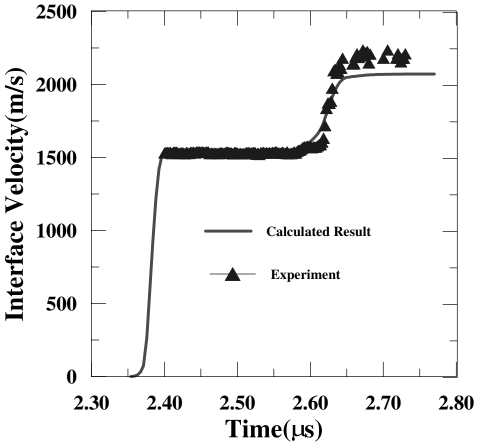

Figure 8. Comparison of the measured u_w–t profile with the calculated one for the shock/reshock experiment on 93W (a 93W/Cu flyer impacts on a 93W sample; impact velocity 2.98 km s^{-1}).

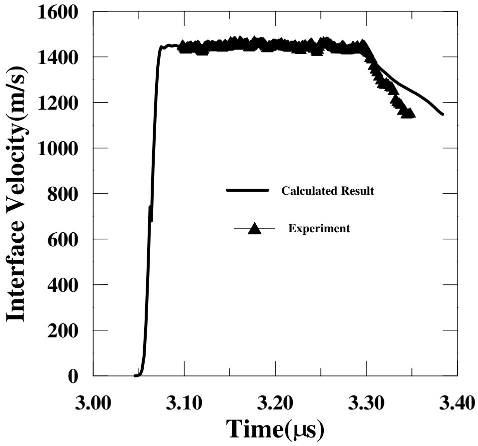

Figure 9. Comparison of the measured u_w–t profile with the calculated one for the shock/reshock experiment on 93W (an Al/93W flyer impacts on a 93W sample; impact velocity 3.46 km s^{-1}).

4. Conclusions

(1) The empirical approximation $Y/G \approx$ constant has been found to be also applicable to shocked 93W at high p and high T, specifically expressed as $Y/G \approx 1.9 \times 10^{-2}$.
(2) When use of the above approximation was incorporated in SCG's constitutive model, we found that equations (1) and (2) can be used with some success to simulate the hydrodynamic behaviour for shocked 93W.
(3) We conjecture that the above approximation, $Y/G \approx$ constant, may be applicable to other metals besides 93W.

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